Exercise 62

When a camera flash goes off, the batteries immediately begin to recharge the flash's capacitor, which stores electric charge given by

$$Q(t) = Q_0(1 - e^{-t/a})$$

(The maximum charge capacity is Q_0 and t is measured in seconds.)

(a) Find the inverse of this function and explain its meaning.

(b) How long does it take to recharge the capacitor to 90% of capacity if $a = 2$?

Solution

To find the inverse, solve the given function for t.

$$Q(t) = Q_0(1 - e^{-t/a})$$

Divide both sides by Q_0.

$$\frac{Q(t)}{Q_0} = 1 - e^{-t/a}$$

Solve for the exponential function.

$$e^{-t/a} = 1 - \frac{Q(t)}{Q_0}$$

Take the natural logarithm of both sides.

$$\ln e^{-t/a} = \ln\left[1 - \frac{Q(t)}{Q_0}\right]$$

Bring the exponent down in front.

$$-\frac{t}{a}\ln e = \ln\left[1 - \frac{Q(t)}{Q_0}\right]$$

Use the fact that $\ln e = 1$.

$$-\frac{t}{a} = \ln\left[1 - \frac{Q(t)}{Q_0}\right]$$

Therefore, the inverse function is

$$t = -a\ln\left[1 - \frac{Q(t)}{Q_0}\right].$$

It gives the input (time) t in terms of the corresponding output (charge) $Q(t)$. One could make it its own function by calling t Q^{-1} and replacing $Q(t)$ with t.

$$Q^{-1} = -a\ln\left(1 - \frac{t}{Q_0}\right).$$

If $a = 2$, then it takes

$$t = -2\ln\left[1 - \frac{0.9Q_0}{Q_0}\right] \approx 4.605 \text{ seconds}$$

for the capacitor to regain 90% of its charge capacity.